

Privacy-Preserving Cryptography from Pairings and Lattices

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Under the supervision of Benoît Libert

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École Normale Supérieure de Lyon, France



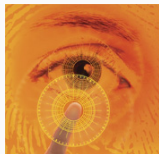
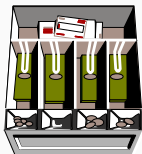
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Allowing functionality while preserving anonymity

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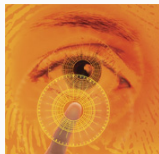
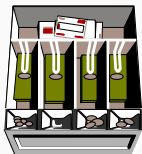
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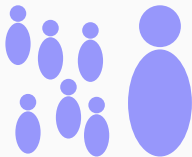
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A user wants to take public transportations.



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▶ Traceability 

Practical group signature
(AsiaCCS'16)

Pairings

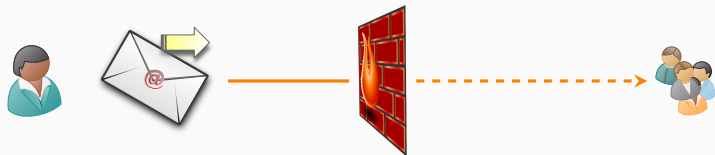
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ZK argument of correct evaluation of committed branching programs
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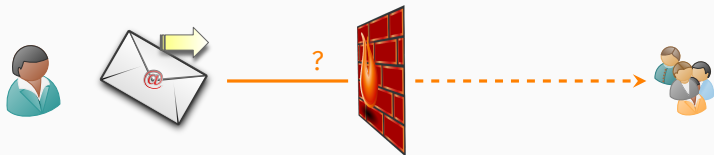
Lattices

Motivation: Firewall Filtering



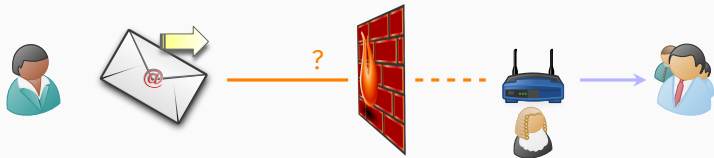
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- ▶ The recipient of the message can be a sensitive information
- ▶ Behind firewall: anonymity is lifted to route messages

History of Group Encryption

2007 Introduction of group encryption (Kiayias-Tsiounis-Yung, Asiacrypt'07)

- ▶ Modular design from anonymous PKE, signatures and interactive ZK proofs
- ▶ Instantiation using number-theoretic assumptions

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- ✗ Existing realizations rely on **quantum-vulnerable** assumptions
- From **lattices**: several realizations of group signatures:
[GKV10, CNR12, LLLS13, NNZ15, LNW15, LLNW16, LMN16, LLMN16]

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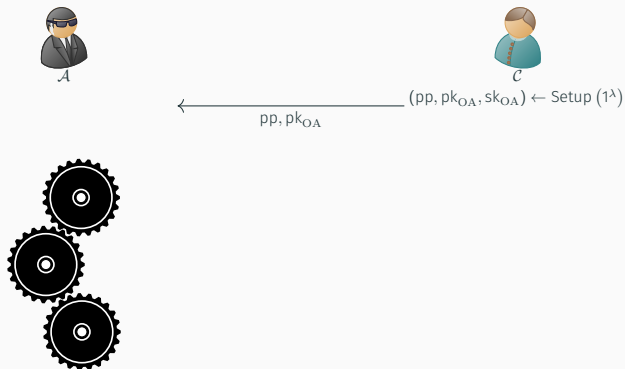
Group Encryption: Receiver Anonymity

Indistinguishability-based game



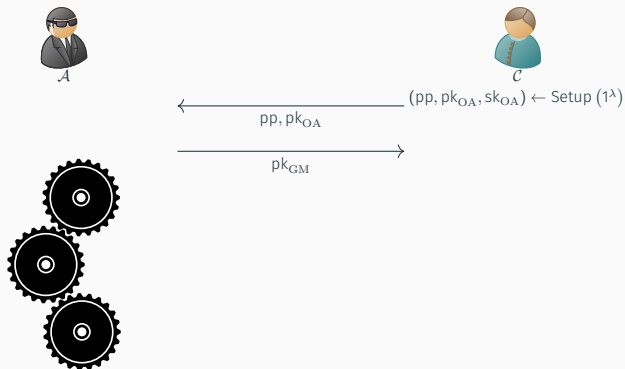
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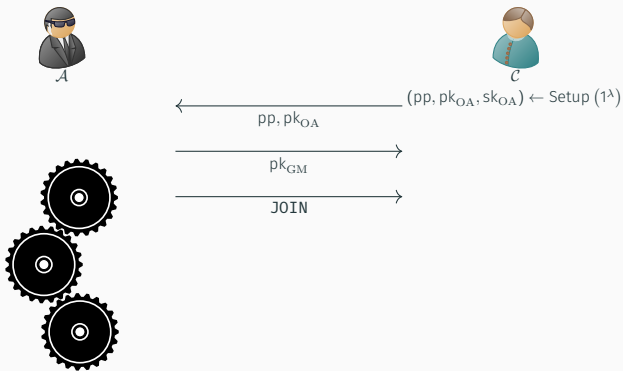
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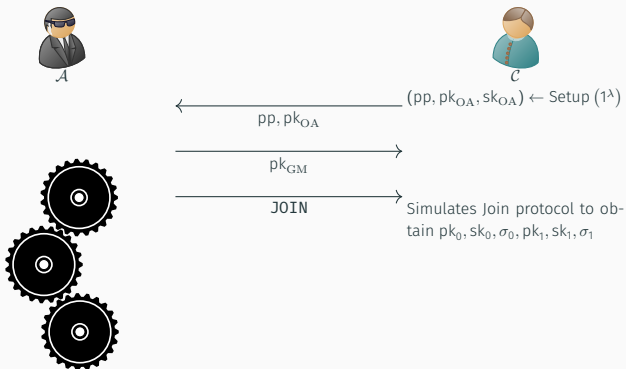
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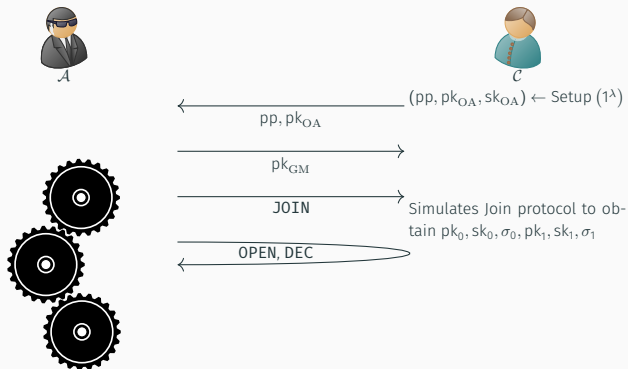
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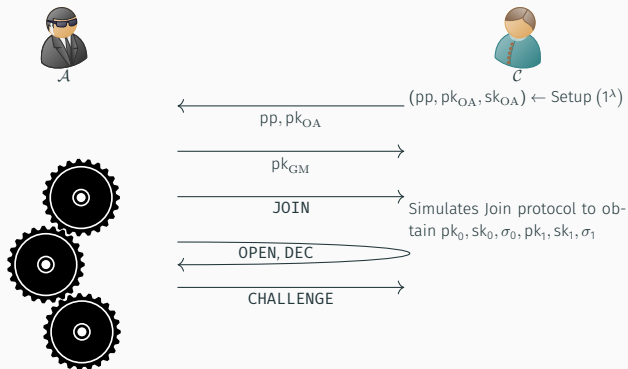
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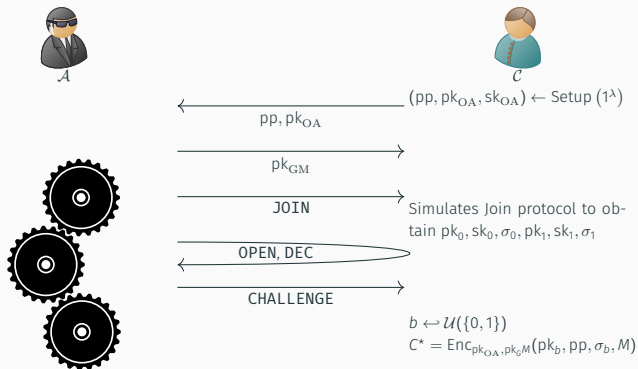
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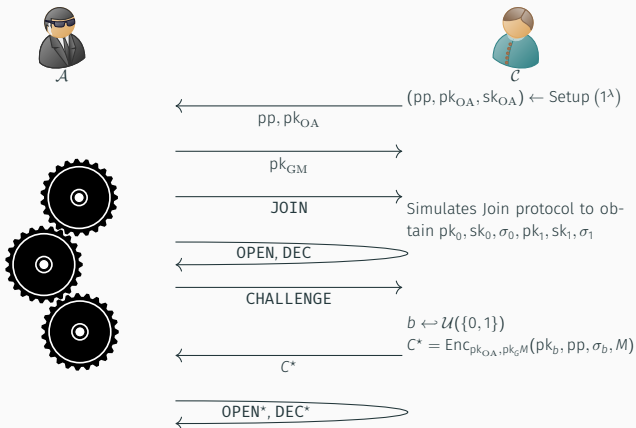
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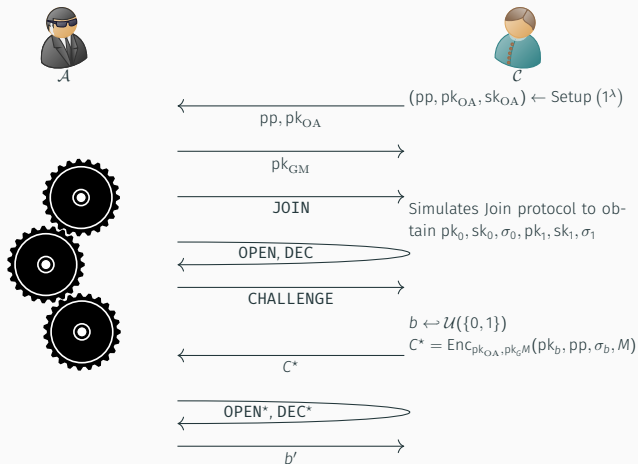
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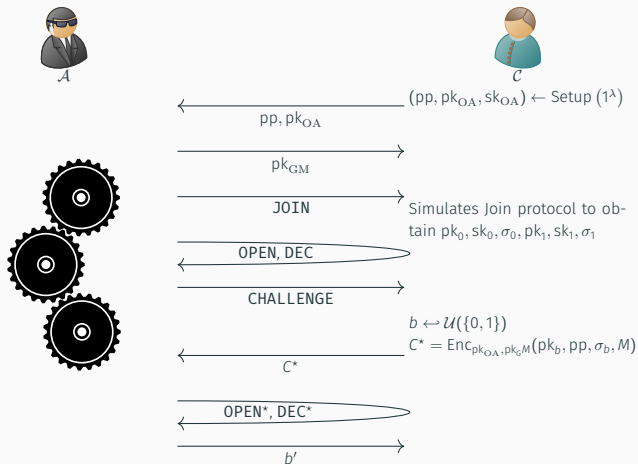
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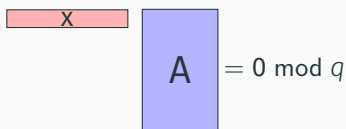
\mathcal{A} wins if $b = b'$

Hardness Assumptions: SIS and LWE (Ajtai 1996, Regev 2005)

Parameters: dimension n , #samples $m \geq n$, modulus q .

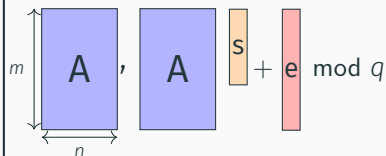
For $A \leftarrow \mathcal{U}(\mathbb{Z}_q^{m \times n})$:

Small Integer Solution


$$Ax = 0 \pmod{q}$$

Goal: Given $A \in \mathbb{Z}_q^{m \times n}$,
find $x \in \mathbb{Z}^m \setminus \{0\}$ small

Learning With Errors


$$As + e \pmod{q}$$

$s \leftarrow \mathbb{Z}_q^n$ e small error

Goal: Given $A, A s + e$,
find $s \in \mathbb{Z}_q^n$

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What is the main difficulty?

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- ▶ Non-interactive variants: NIZK proofs
- ▶ Random Oracle: allows transforming ZK to NIZK (Fiat-Shamir, Crypto'86)
- ▶ Standard Model: using bilinear maps (Groth-Sahai, Eurocrypt'08)

Zero-Knowledge Proofs for Lattices

Two main proof systems in lattice-based cryptography:

Schnorr-like (Crypto'89): On Ring-LWE¹, concise but not expressive.

Stern-like (Crypto'93): On LWE², heavy but expressive.

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Examples: (I)SIS and LWE relations are linear

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Modular design for GS¹: sign-then-encrypt-then-prove

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Remark: The message is embedded in the NIZK proof

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Main Difficulty in Group Encryption

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X We have to handle relations with **hidden-but-certified** matrix:

$$\boxed{x} \cdot \boxed{s} + \boxed{e} = \boxed{b} \pmod{q}$$

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Syndrome Decoding Problem

Given $\mathbf{P} \in \mathbb{Z}_2^{n \times m}$ and $\mathbf{v} \in \mathbb{Z}_2^n$, find \mathbf{x} s.t. $w(\mathbf{x}) = w$ and

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Kawachi-Tanaka-Xagawa'08: $\text{mod } 2 \rightarrow \text{mod } q$

Ling-Nguyen-Stehlé-Wang'13: Extends Stern's protocol to SIS/LWE

Recent uses of Stern-like protocols in lattice-based crypto:

[LNW15, LLNW16, LMN16, LLNMW16, LLNMW17, LLNW17]

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Idea:

1. **Pre-process** the given quadratic relation
2. Exploit **permutations** to prove the relation

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Goal: Express $X \cdot S$ as $Q \cdot Z$

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2. $x_i \cdot s_i = H \cdot (x_{i,1} \cdot s_i, \dots, x_{i,mk} \cdot s_i)^T$

$k = \lceil \log q \rceil$; H is the decomposition matrix $\mathbb{Z}_q^m \rightarrow \{0, 1\}^{mk}$

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Goal: Express $\boxed{X} \cdot \boxed{s}$ as $\boxed{Q} \cdot \boxed{z}$

Idea: Binary decomposition

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x_i denotes i -th column of X ; s_i is the i -th component of s

2. $x_i \cdot s_i = H \cdot (x_{i,1} \cdot s_i, \dots, x_{i,mk} \cdot s_i)^T$

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$x_{i,j} \cdot s_i$ has form “(public matrix)·(secret vector)” \Rightarrow so does $x_i \cdot s_i$

\Rightarrow so does $\boxed{X} \cdot \boxed{S} = \boxed{Q} \cdot \boxed{Z} \pmod q$

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Prove that a secret bit z is of form $z = c_1 \cdot c_2$, while remaining able to prove that the c_1 and c_2 satisfy other equations.

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$$\text{ext}(c_1, c_2) = (\bar{c}_1 \cdot \bar{c}_2, \bar{c}_1 \cdot c_2, c_1 \cdot \bar{c}_2, c_1 \cdot c_2)^T$$

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Note that for all $c_1, c_2, b_1, b_2 \in \{0, 1\}$, it holds that

$$\mathbf{v} = \text{ext}(c_1, c_2) \iff T_{b_1, b_2}(\mathbf{v}) = \text{ext}(c_1 \oplus b_1, c_2 \oplus b_2)$$

Solution to the Sub-Problem

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- ▶ same bits c_1, c_2 appear in other equations \Rightarrow same masks b_1, b_2

Group Encryption: Putting Everything Together

Ingredients

- ▶ Anonymous encryption [ABB10] IBE + [CHK04] transform
- ▶ Signature scheme [LLMNW16]
- ▶ Supporting ZK proofs [Presented result]
- + Modular construction [KTY07]

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- ▶ Our results (Libert-Ling-M-Nguyen-Wang, Asiacrypt'16):

- Zero-Knowledge arguments for “quadratic relations”:

$$x \cdot s + e = b \pmod{q}$$

→ Building block for cryptography: may be of independent interest

- First construction of group encryption from (classical) lattice assumptions proven secure in the standard model

Practical group signature
(AsiaCCS'16)

Pairings

First lattice-based signature with efficient protocols
(Asiacrypt'16)

ZK argument of correct evaluation of committed branching programs
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ZK argument for quadratic relations
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Lattices

First Lattice-Based Signature with Efficient Protocols

(Libert-Ling-M-Nguyen-Wang, Asiacrypt'16)

A signature scheme (Keygen , Sign_{sk} , Verif_{vk}) with efficient protocols¹:

- ▶ To **sign** a committed value;
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✗ Existing constructions rely on Strong RSA assumption or bilinear maps.

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Adaptive Oblivious Transfer

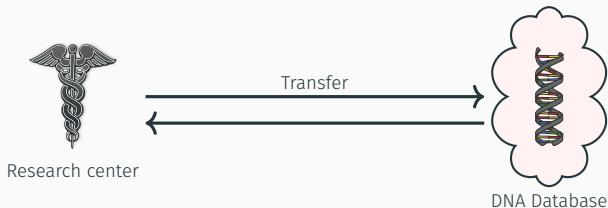
(Naor-Pinkas, Crypto'99; Libert-Ling-M-Nguyen-Wang, Asiacrypt'17)



- ▶ DNA storage is expensive

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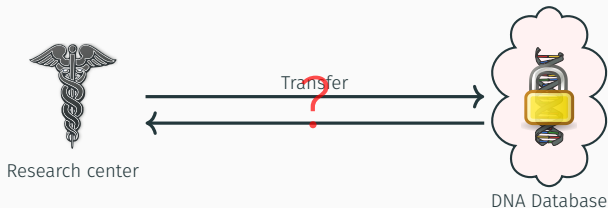
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Extending expressiveness of Stern-like protocols

⇒ First construction from lattices with access control

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Practical Group Signatures from Pairing Assumptions

(Libert-M-Peters-Yung, AsiaCCS'16)

Pairing

Let \mathbb{G} , $\hat{\mathbb{G}}$ and \mathbb{G}_T be cyclic groups of prime order p .

$$e : \mathbb{G} \times \hat{\mathbb{G}} \longrightarrow \mathbb{G}_T$$

$$\forall g \in \mathbb{G}, \hat{g} \in \hat{\mathbb{G}}, a, b \in \mathbb{Z}, e(g^a, \hat{g}^b) = e(g, \hat{g})^{ab}$$

Hardness relies on (variant of) *Decision Diffie-Hellman*

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- + Pairings are more practical than lattices
- ▶ Design supported by an open-source implementation in C
- ▶ Use Relic toolkit 

<https://gforge.inria.fr/projects/sigma-sig-c/>

My Research so far

Around two axes:

- ▶ (Privacy-preserving) protocol design
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Stern-like proofs Constant soundness error of $2/3$

Follow-ups

- ▶ Universally composable oblivious transfer from LWE?
- ▶ More efficient compact e-cash system?

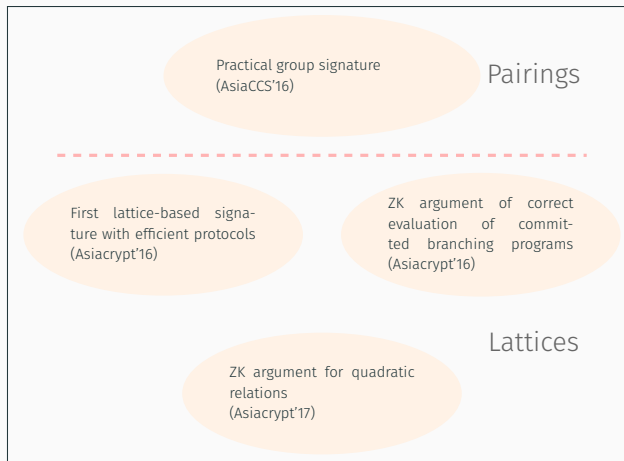
Zero-knowledge proofs

- ▶ Negligible soundness error for expressive statements in lattices?
- ▶ NIZK for NP from LWE?

Cryptographic constructions

- ▶ More efficient signatures (compatible with ZK proofs)?
- ▶ Efficient trapdoor-free (H)IBE?

Thank you for your Attention



What next? More protocol designs, zero-knowledge proofs and foundations of cryptographic constructions!