Zero-Knowledge Arguments for Matrix-Vector Relations and Lattice-Based Group Encryption

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Privacy-Preserving Cryptography

Goal: Provide functionalities while keeping users anonymous

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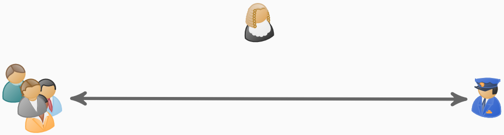

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Examples: Group Signatures, Anonymous Credentials, e-Cash, ...

Main ingredients:

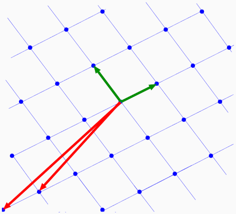
- Digital signatures;
- Public-Key encryption;
- Supporting Zero-Knowledge proofs.

Lattice-Based Cryptography [Ajt96, Reg05]

Lattice

A lattice is a discrete subgroup of $\mathbb{R}^n$. Can be seen as integer linear combinations of a finite set of vectors.

$$\Lambda(\mathbf{b}_1, \ldots, \mathbf{b}_n) = \left\{ \sum_{i \leq n} a_i \mathbf{b}_i \mid a_i \in \mathbb{Z} \right\}$$



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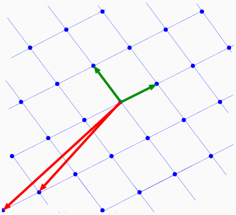


Why?

- Simple and asymptotically efficient;
- **Still** conjectured quantum-resistant;
- Connection between average-case and worst-case problems;
- Powerful functionalities (e.g., FHE).

$\rightarrow$ Finding a short non-zero vector in a lattice is hard.

Hardness Assumptions: SIS and LWE [Ajt96, Reg05]

Small Integer Solution

$$\mathbf{x} \, \mathbf{A} = \mathbf{0} \ [q]$$

Goal: Given $\mathbf{A} \in \mathbb{Z}_q^{m \times n}$, find $\mathbf{x} \in \mathbb{Z}^m \backslash \{\mathbf{0}\}$ small

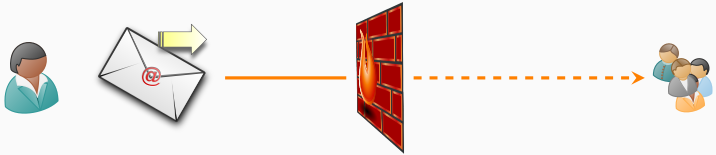
Learning With Errors

$$\left(\underbrace{\mathbf{A}}_{n} \Big\}^{m} , \; \mathbf{A}\,\mathbf{s} + \mathbf{e} \right)$$

$\mathbf{s} \hookleftarrow \mathbb{Z}_q^n$ $\quad$ $\mathbf{e}$ small error

Goal: Given $\left(\mathbf{A}, \mathbf{A}\,\mathbf{s} + \mathbf{e}\right)$, find $\mathbf{s} \in \mathbb{Z}_q^n$

Firewall Filtering [LYJP14]



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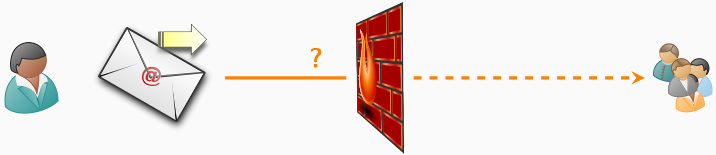

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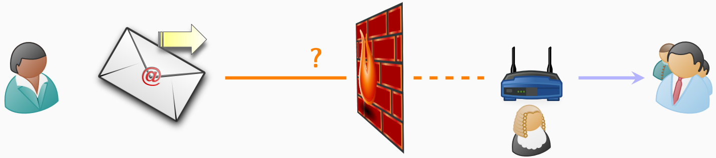

- A user wants to send a message to a group behind a firewall
- The recipient of the message can be a sensitive information
- The router can lift anonymity to route messages

Group Encryption [KTY07]

Group encryption allows encrypting while proving that:

1. The ciphertext is well-formed and intended for some registered group member who will be able to decrypt;
2. The opening authority will be able identify the receiver if necessary;
3. The plaintext satisfies certain properties.

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Possible applications

- Firewall filtering
- Anonymous trusted third parties
- Cloud storage services
- Hierarchical group signatures [TW05]

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Accountability

Group members are kept accountable for their actions: an opening authority can un-anonymize the signatures/ciphertexts if necessary. needs arise.

History

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✗ All existing realizations of GE rely on number-theoretic assumptions

? Construction from other assumptions, e.g., lattice-based?

Outline

Introduction

Toward Realizing Group Encryption

Zero-Knowledge Arguments for Group Encryption

In the Lattice-Based World

Remark

Group signatures and group encryption rely on the same building blocks

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? What is the main difficulty?

Zero-Knowledge Proofs for Lattices

Two main proof systems in lattice-based cryptography:

Schnorr-like [Sch89]: On Ring-LWE [Lyu08], concise but not expressive.
Algebraic

Stern-like [Ste93]: On LWE, heavy but expressive.
Combinatorial

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Examples: (I)**SIS** and **LWE** relations are linear

The Case of Group Signatures

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✓ Known techniques allow realizing the ZK proofs

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  2. Sender knows a valid signature σ on pk, w.r.t. $\mathsf{vk}_{\mathrm{GM}}$; $\mathbf{c}_{\mathrm{OA}}$ is a correct encryption of pk, w.r.t. $\mathsf{pk}_{\mathrm{OA}}$; μ satisfies $\mathcal{R}$.

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We have to handle relations with **hidden-but-certified** matrix:

$$\mathbf{X} \cdot \mathbf{s} + \mathbf{e} = \mathbf{b} \bmod q$$

We call this "quadratic relations".

Zero-Knowledge Arguments for Group Encryption

Stern's Protocol [Ste93]

Stern's protocol is a ZK proof for Syndrome Decoding Problem.

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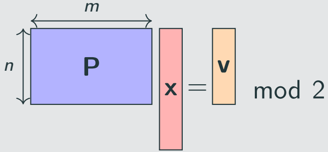
Syndrome Decoding Problem

Given $\mathbf{P} \in \mathbb{Z}_2^{n \times m}$ and $\mathbf{v} \in \mathbb{Z}_2^n$, find $\mathbf{x}$ s.t. $\mathrm{w}(\mathbf{x}) = w$ and



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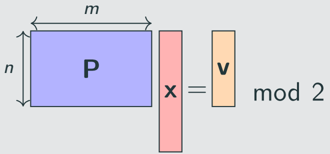

[KTX08]: mod $2 \rightarrow$ mod q

[LNSW13]: Extends Stern's protocol for SIS and LWE statements

Recent uses of Stern-like protocols in lattice-based crypto:
[LNW15, LLNW16, LMN16, LLNMW16, LLNMW17, LLNW17]

Stern's Ideas

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1. **Permuting:** Proving the witness constraint using random permutation
   - Send the verifier $\pi(\mathbf{x})$
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2. **Masking:** Proving linear equation using a random mask $\mathbf{r}$
   - Send the verifier $\mathbf{y} = \mathbf{x} + \mathbf{r}$ and show that $\mathbf{P} \cdot \mathbf{y} = \mathbf{v} + \mathbf{P} \cdot \mathbf{r}$

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Idea: We will

2.1 Pre-process the given quadratic relation

2.2 Exploit permuting to prove the relation

Dealing with Quadratic Relations: First Step

Goal: Express $\mathbf{X} \cdot \mathbf{s}$ as $\mathbf{Q} \cdot \mathbf{z}$

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Goal

Prove that a secret bit z is of form $z = c_1 \cdot c_2$, while preserving the possibility to show that the component bits c_1 and c_2 satisfy other equations.

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Define bit-extension $\mathrm{ext}(\cdot, \cdot)$ and permutation $T_{b_1, b_2}(\cdot)$ s.t.

$$\mathbf{v} = \mathrm{ext}(c_1, c_2) \iff T_{b_1, b_2}(\mathbf{v}) = \mathrm{ext}(c_1 \oplus b_1, c_2 \oplus b_2)$$

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2. Permute $\mathbf{v}$ with random bits b_1, b_2, and give the verifier the permuted vector $T_{b_1,b_2}(\mathbf{v})$
3. To prove that the **same** bits c_1, c_2 appear in other equations: set up similar mechanisms at their other appearances, and use the **same** b_1, b_2.

Group Encryption: Putting Everything Together

Ingredients

- Anonymous encryption — [ABB10] IBE + [CHK04] transform
- Signature scheme — [LLMNW16]
- Supporting ZK proofs — [This Talk]

[KTY07]'s modular construction $\Rightarrow$ first group encryption construction from (classical) lattice assumptions proven secure in the standard model

Conclusion

- Our results:
  - Zero-Knowledge arguments for "quadratic relations":

$$\mathbf{X} \cdot \mathbf{s} + \mathbf{e} = \mathbf{b} \bmod q.$$

  $\rightarrow$ Building block for cryptography: may be of independent interest
  - First lattice-based group encryption scheme

Questions?