A Lattice-Based Group Signature Scheme with Message-Dependent Opening

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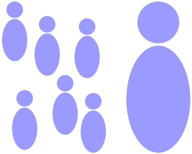
ACNS, Guildford — June 20th, 2016

Example

Public Transportation

Some user wants to take public transportations.



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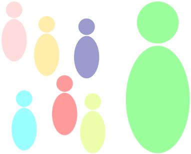

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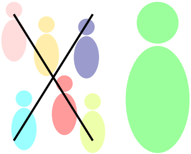

signature

- Authenticity & Integrity
- Anonymity
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- Avoid opening abuses

Message Dependent Opening

(Sakai et al. Pairing'12)

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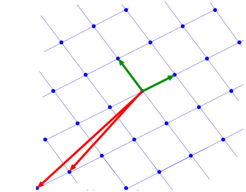
$\rightarrow$ **Idea:** Add another authority to restrict the power of the OA.

The admitter delivers tokens that allow OA to open all signatures for specific messages.

Lattice-Based Cryptography

A **Lattice** is the set of integer linear combination of independent vectors called a basis

$$\Lambda(\mathbf{b}_1, \ldots, \mathbf{b}_n) = \left\{ \sum_{i \leq n} a_i \mathbf{b}_i \;\middle|\; \forall i, a_i \in \mathbb{Z} \right\}$$



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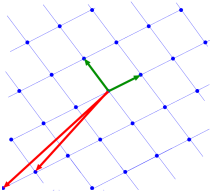

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Simple, efficient, conjectured resistant to a quantum adversary, links between average-case and wost-case problems, expressive...

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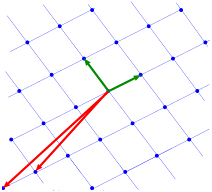

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Average-case problems: SIS and LWE

Outline

State of the art

- Introduction by Chaum and van Heyst (Eurocrypt'91)
- First scalable solution.
  Ateniese-Camenisch-Joye-Tsudik (Crypto'00)
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- Scheme in the standard model. Libert-Joye (CT-RSA'14)
- Efficient lattice-based signature (LNW and NZZ PKC'15)

No lattice-based GS-MDO so far

GPV IBE

(Gentry-Peikert-Vaikuntanathan; STOC'08)

Identity Based Encryption: To encrypt $\mathbf{m} \in \{0,1\}^\ell$ under id

Setup Generate $\mathbf{A} \in \mathbb{Z}_q^{n \times m}$ with a trapdoor $\mathbf{T_A} \in \mathbb{Z}^{m \times m}$

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Extract Let $\mathbf{G} = \mathcal{H}(\mathsf{id})$.
Use $\mathbf{T_A}$ compute small norm matrix $\mathbf{E} \in \mathbb{Z}^{m\times \ell}$ s.t.

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Encrypt Sample $(\mathbf{s}, \mathbf{e}_1, \mathbf{e}_2) \hookleftarrow \chi^n \times \chi^m \times \chi^{\ell}$ and output

$$\mathbf{c} = \left(\mathbf{A}^T \cdot \mathbf{s} + \mathbf{e}_1, \mathbf{G}^T \cdot \mathbf{s} + \mathbf{e}_2 + \lfloor q/2 \rfloor \cdot \mathbf{m}\right).$$

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Decrypt $\mathbf{m}' = \boxed{1 \mid 2 \mid 4 \mid \cdots \mid 2^{\ell-1}} \cdot \left\lfloor (\mathbf{c}_2 - \mathbf{E}^T \cdot \mathbf{c}_1) \cdot (q/2) \right\rceil$

Boyen's Signature

(PKC'10)

To sign a message $M = m_1 \cdots m_\ell \in \{0,1\}^\ell$:

KeyGen: Generate matrix $\mathbf{A}$ with trapdoor $\mathbf{T_A}$, random matrices $\mathbf{A}_0, \ldots, \mathbf{A}_\ell \in \mathbb{Z}_q^{n \times m}$ and a vector $\mathbf{u} \in \mathbb{Z}_q^n$.

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Sign: Using $\mathbf{T}_\mathbf{A}$, compute short $\mathbf{z} \in \mathbb{Z}^{2m} = \sigma$ s.t.

$$\underbrace{\left[\, \mathbf{A} \;\middle|\; \mathbf{A}_0 + \sum_{i=1}^{\ell} m_i \cdot \mathbf{A}_i \,\right]}_{\mathbf{A}_M} \mathbf{z} = \mathbf{u} \qquad (*)$$

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Verify: Test $\|\mathbf{z}\| \leq \beta$.
Compute $\mathbf{A}_M$ to check relation $(*)$.

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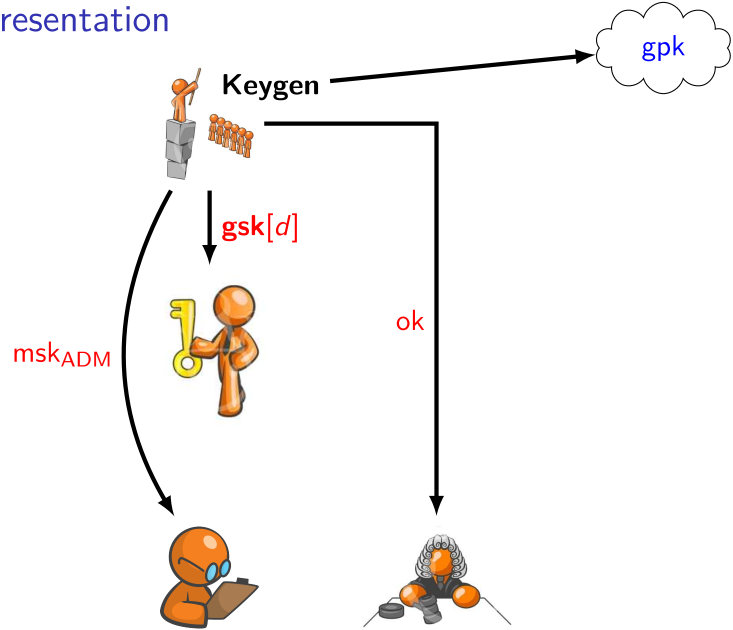
[KTX08]: mod 2 $\rightarrow$ mod q

[LNSW13]: Extend Stern's protocol for SIS and LWE statements

Recent uses of Stern-like protocols in lattice-based crypto:
[LNW15], [LLNW16], [LLNMW16]

Outline

Presentation



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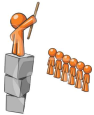
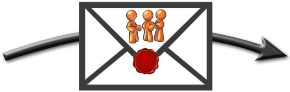


Sign

gsk[d]

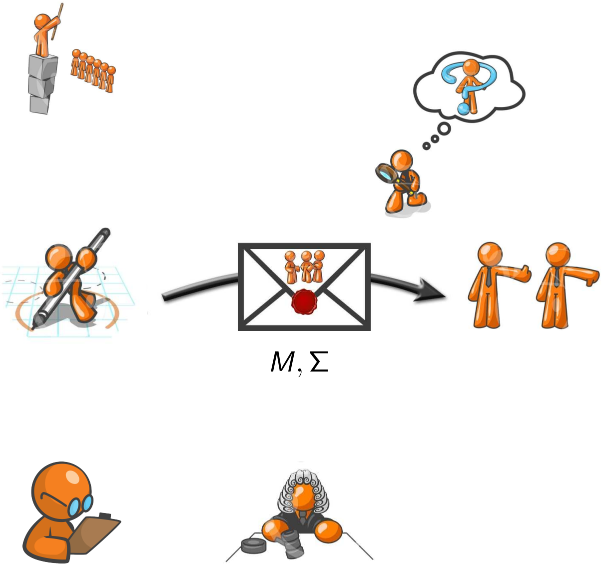


M, Σ

Verify

gpk

Presentation



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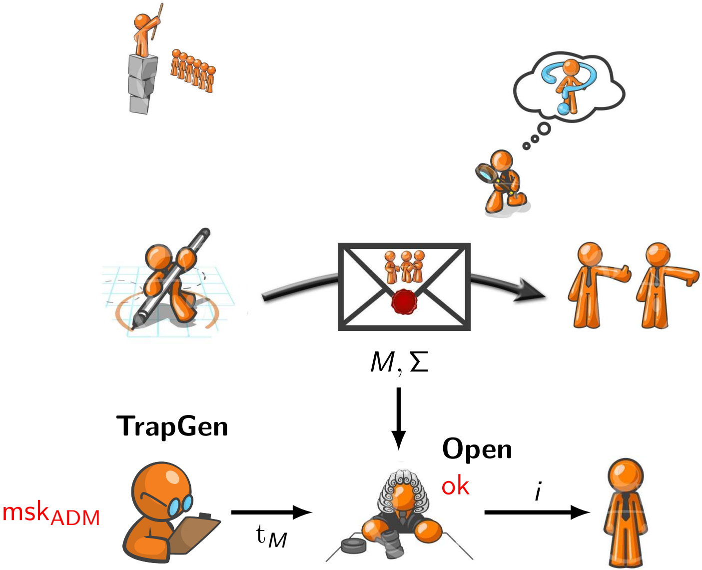

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  `Input:` security parameter λ and group size N
  `Output:` public parameters gpk, opening authority's secret key ok admitter's master secret key $\mathsf{msk}_{\mathsf{ADM}}$,

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- **Open:**

  `Input:` **OA**'s secret ok, a token t_M, message M and Σ
  `Output:` identity i or error $\perp$

Generic Construction of Group Signatures

(Bellare-Micciancio-Warinschi; Eurocrypt'03)

E is an encryption scheme, S is a signature scheme.

- **Keygen.** $1^\lambda, 1^N \to \mathbf{gsk}, \mathrm{ok}, \mathrm{gpk}$
  - $(S.vk, S.sk) \leftarrow S.Keygen$
  - $\mathbf{gsk}[d] \leftarrow S.Sign_{S.sk}(d)$ for $d = 1, \ldots, N$
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- **Sign:** $d, \mathbf{gsk}[d], M \rightarrow (C, \pi)$
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- **Open:** $(C, \pi), M, \mathrm{ok} \rightarrow \{1, \ldots, N\} \cup \perp$
  - Decrypt C with $\mathrm{ok} = E.sk$

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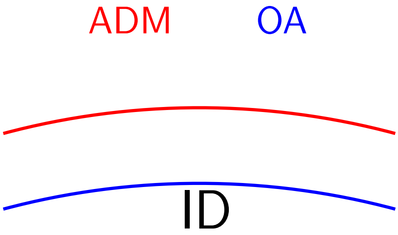

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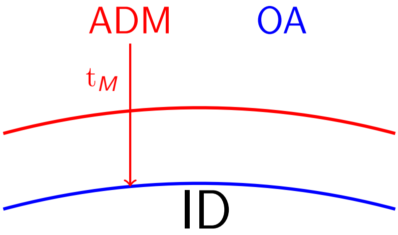

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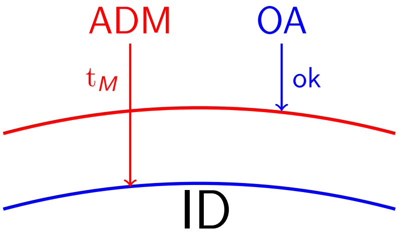

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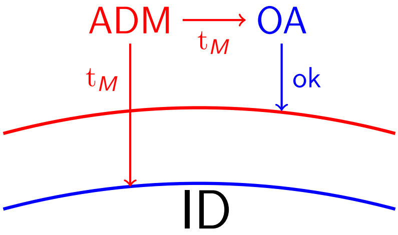

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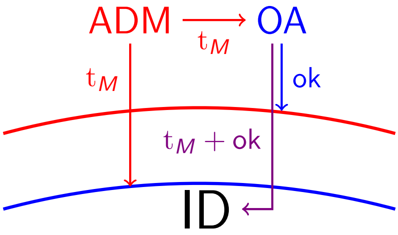

General Construction

(Sakai et al. Pairing'12)

- **KeyGen:**
  - Generate keys $(\mathsf{pk}, \mathsf{sk})$ for signature and

$$\mathbf{gsk}[d] = \mathsf{Sig.Sign}_{\mathsf{sk}}(d)$$

  - Generate two key pairs for the GPV IBE:

$$(\mathbf{B}, \mathbf{T_B}) \text{ and } (\mathbf{C}, \mathbf{T_C})$$

  - Output

$$\mathsf{ok} = \mathbf{T_B} \qquad \mathsf{msk}_{\mathsf{ADM}} = \mathbf{T_C}$$

$$\mathbf{gsk} \qquad \mathsf{gpk} = (\mathsf{pk}, \mathbf{B}, \mathbf{C}, \mathsf{OTS}, \mathcal{H})$$

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- **Sign:** $d, M \mapsto$
  - $C \leftarrow Enc(d)$, using CHK
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  - Prove possession of $(d, \mathbf{gsk}[d])$ and that everything is correct

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- **Open:**
  - $C \leftarrow IBE.Dec_{\mathrm{t}_M}(\widehat{C})$
  - $d \leftarrow Dec(C)$

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We need to prove double-encryption relations in ZK

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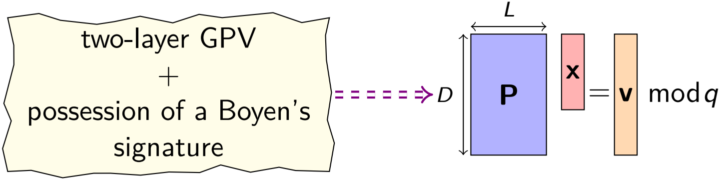
- Possible because relations can be transformed into



with $\mathbf{x} \in \{-1, 0, 1\}^L$ and $\mathbf{v} \in \mathbb{Z}_q^D$.

Conclusion

- We provide:
  - A lattice-based group signature scheme with message dependent opening
  - Security in the ROM under standard lattice assumptions
  - A modular technique that extends [LNW15]
- We can easily adapt the technique of [LLMNW16] for dynamic group signatures to get message-dependent openings for dynamic groups

Questions?