Practical "Signatures with Efficient Protocols" from Simple Assumptions

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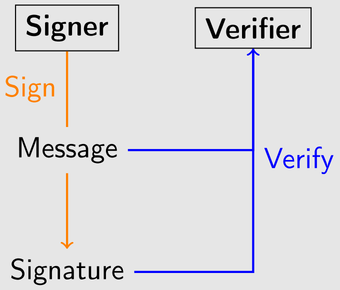
Digital Signatures

Signature Scheme



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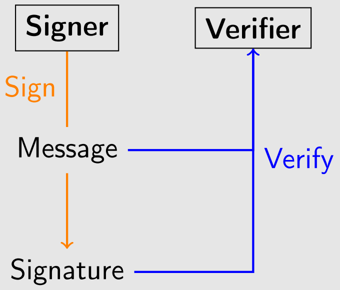
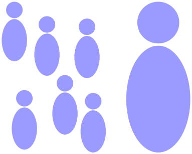


Guarantees authenticity and integrity.

Group Signatures

Bob wants to take public transportations.



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Group Signatures

Bob wants to take public transportations.

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- Traceability

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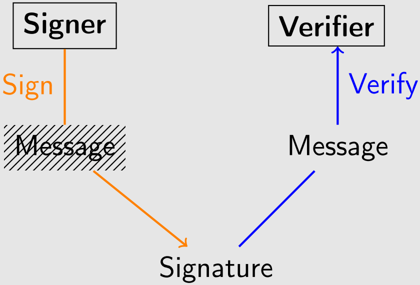
- **Binding** property: once sealed, a value cannot be changed
- **Hiding** property: nobody can tell what is inside the box without the key

Signature with Efficient Protocols

Signature Scheme with Efficient Protocols (Camenisch-Lysyanskya, SCN'02)



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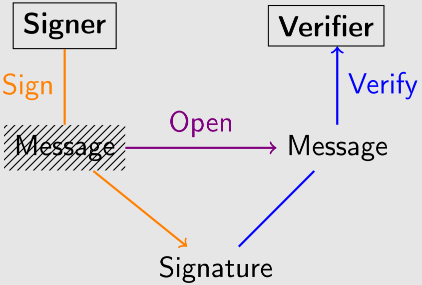

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- Sign committed values

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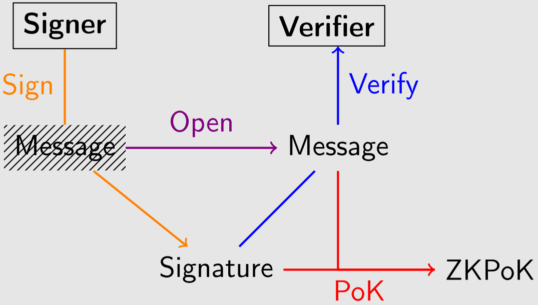

- Signature
- Sign committed values
- Proof of Knowledge (PoK) of (Message; Signature)

Pairing-Based Cryptography

Pairing

$$e : \mathbb{G} \times \hat{\mathbb{G}} \longrightarrow \mathbb{G}_T$$

s.t. for $g \in \mathbb{G}, \hat{g} \in \hat{\mathbb{G}}$

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Hardness assumptions:

- **SXDH**: *DDH* holds in $\mathbb{G}$ and $\hat{\mathbb{G}}$ with $\mathbb{G} \neq \hat{\mathbb{G}}$
  - **DDH**: given (g, g^a, g^b, g^c), tells whether $c = a \cdot b$ or $c \in_R \mathbb{Z}_p$
- **SDL**: given $(g, \hat{g}, g^a, \hat{g}^a)$, compute $a \in \mathbb{Z}_p$ with $p = |\mathbb{G}|$

$\rightarrow$ Well studied, fixed-size assumptions.

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- **DDH**: $(g^a, g^b, g^c) \mapsto$ tells if $c = ab$ or c random.

***q*-type** assumptions

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Non-interactive assumptions

- **DL**: $g^a \in \mathbb{G} \mapsto a \in \mathbb{Z}_p$

Interactive assumptions

- **One-more-DL**: given oracle access to $(g^{a_i} \mapsto a_i)$, finds $(b_i)_i$ given $(g^{b_i})_i$

Outline

Signature

Signature Scheme:

- Constant-size

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- Sign committed messages
- ZK-Prove the knowledge of a valid message-signature pair

Linear Subspace Membership

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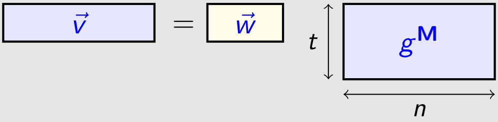
We say that $\vec{v} \in \mathrm{Span}(\mathrm{Rows}(\mathbf{M}))$ if there exists $\vec{w} \in \mathbb{Z}_p^t$ satisfying

$$\vec{v} = g^{\vec{w} \cdot \mathbf{M}} \in \mathbb{G}^n$$



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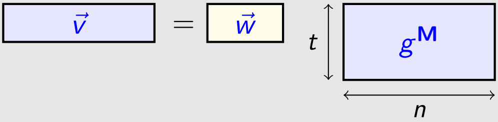

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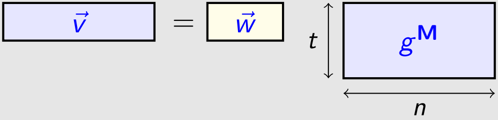

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Quasi-Adaptive (Jutla-Roy (Asiacrypt'13)) means that the *common reference string* (*crs*) may depend on the language (here the matrix $\mathbf{M}$)

Proof System for Linear Subspace Membership

Use of Kiltz-Wee Quasi-Adaptive Non-Interactive ZK proofs (QA-NIZK) to prove linear subspace membership.

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Kiltz-Wee QA-NIZK (Eurocrypt'15)

Given $\mathbf{M} = (\vec{M}_1, \ldots, \vec{M}_t)^T \in \mathbb{G}^{t \times n}$,

$\pi \in \mathbb{G}$ prove that $\vec{v} \in \mathrm{Span}(\mathrm{Rows}(\mathbf{M}))$ for some witness $\vec{w}$.

Which is constant-size.

Our Signature Scheme

$$pk = (\mathsf{cp}, \mathsf{crs}, \vec{v} = (v_1, \ldots, v_\ell, w) \in_R \mathbb{G}^{\ell+1}, \Omega = h^\omega) \quad sk = \omega$$

$$\mathbf{M} = \begin{pmatrix} g & 1 & \cdots & 1 & 1 & 1 & 1 & \ldots & 1 & h \\ v_1 & g & 0 & \cdots & 0 & h & 0 & \cdots & 0 & 1 \\ \vdots & 0 & \ddots & 0 & \vdots & 0 & \ddots & 0 & \vdots & \vdots \\ v_\ell & 0 & \cdots & g & 0 & 0 & \cdots & h & 0 & 1 \\ w & 0 & \cdots & 0 & g & 0 & \cdots & 0 & h & 1 \end{pmatrix}$$

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$$\sigma_1 = g^\omega (v_1^{m_1} \cdots v_\ell^{m_\ell} w)^s \qquad \sigma_2 = g^s \qquad \sigma_3 = h^s$$

$+\ \pi$: ZK proof that

$$(\sigma_1, \sigma_2^{m_1}, \ldots, \sigma_2^{m_\ell}, \sigma_2, \sigma_3^{m_1}, \ldots, \sigma_3^{m_\ell}, \sigma_3, \Omega) \in \mathrm{Span}(\mathrm{Rows}(\mathbf{M}))$$

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The signature scheme is secure under **chosen-message attack** under **SXDH**.

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- **Setup:** done by a trusted entity

  `Input`: security parameter λ, bound on group size N
  `Output`: public parameters $\mathcal{Y}$, group manager's secret key $\mathcal{S}_{\mathsf{GM}}$, the opening authority's secret key $\mathcal{S}_{\mathsf{OA}}$

Definition

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- **Join:** interactive protocols between $\mathcal{U}_i \rightleftarrows$ **GM**.

  Provides $(\mathsf{cert}_i, \mathsf{sec}_i)$ to $\mathcal{U}_i$.

  Where cert_i attests the secret sec_i.

  Updates the list of users and membership certificates.

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Dynamic Group Signature

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- **Sign** and **Verify** proceed as in standard digital signatures
- **Open:**

  `Input:` **OA**'s secret $\mathcal{S}_{\mathsf{OA}}$, M and Σ
  `Output:` i or $\perp$

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Three security notions

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$\rightarrow$ Decryption queries correspond to opening queries

Generic Construction

- **Keygen** $\rightarrow \mathcal{S}_{\mathsf{GM}}, \mathcal{S}_{\mathsf{OA}}, \mathcal{Y}$
  - $\mathcal{S}_{\mathsf{GM}} \leftarrow$ Sign.sk
  - $\mathcal{S}_{\mathsf{OA}} \leftarrow$ Enc.sk
  - $\mathcal{Y} \leftarrow$ (Sign.pk, Enc.pk)

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  - $\widetilde{\mathsf{cert}} \leftarrow \mathcal{U}_i$ re-randomize cert_i
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Use of the previous signature with efficient protocols.

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Name	Signature length			Assumptions	Group Type	Anonymity
	$\mathbb{G}$	$\mathbb{Z}_p$	bitsize			
BBS04	3	6	2 304	q-SDH + DLIN	Static	CPA
DP06	4	5	2 304	q-SDH + XDH	Dynamic	CCA
BCNSW10	3	2	1 280	*interactive* + SDL	Dynamic	CCA-
PS16	2	2	1 024	*interactive*	Dynamic	CCA-
Ours	7	3	2 560	SXDH + SDL	Dynamic	CCA

Table: Comparison between different group signature schemes

CCA- means selfless-CCA-anonymity

Conclusion

We propose:

- A group signature built on **well studied assumptions** with comparable signature length with other schemes
  - Almost as efficient as Delerablée-Pointcheval'06
- A rather efficient signature with efficient protocols that can be used for other privacy-friendly protocols
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Any Question?