Designing a Dynamic Group Signature Scheme using Lattices

M2 Internship Defense

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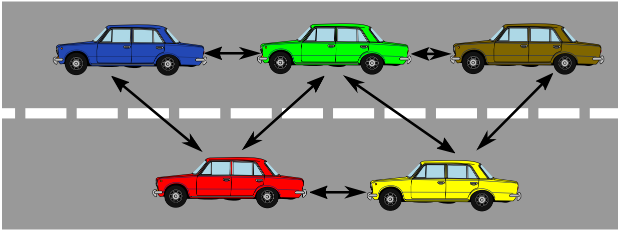
ENS DE LYON

Example

Smart cars



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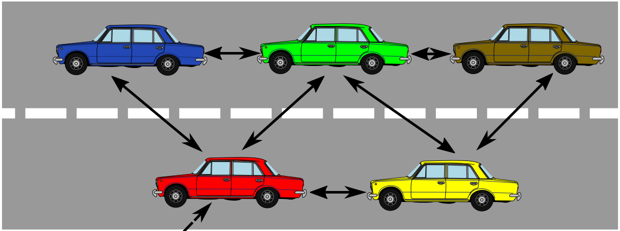
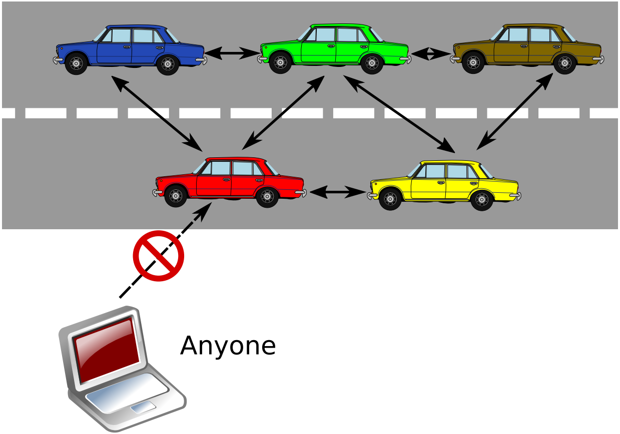


Anyone

Example

Smart cars

- Authenticity
- Integrity



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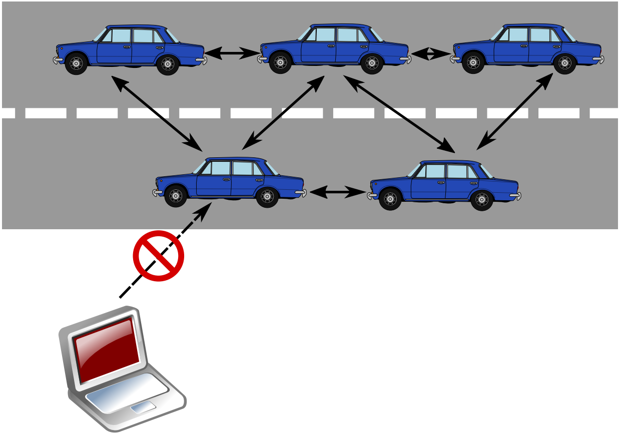

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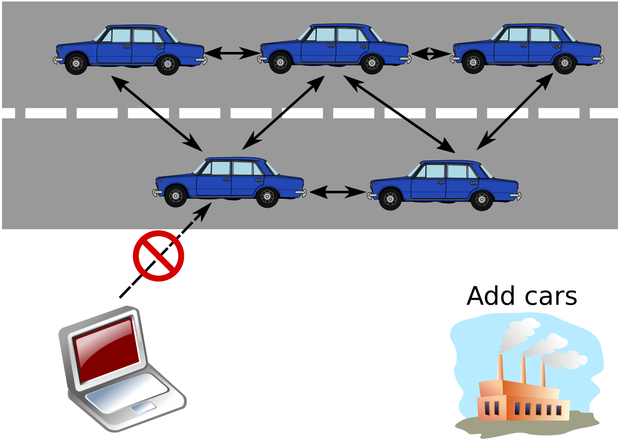

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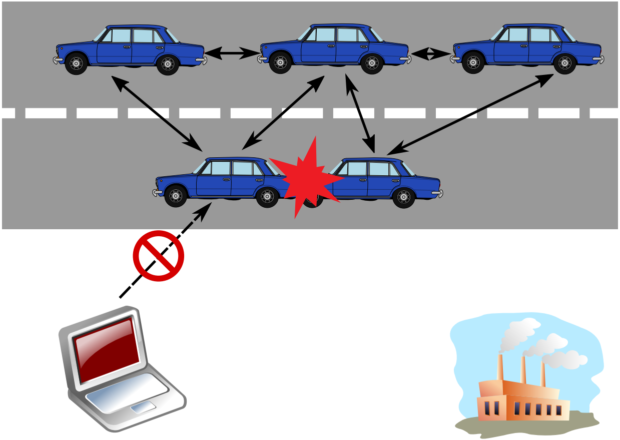
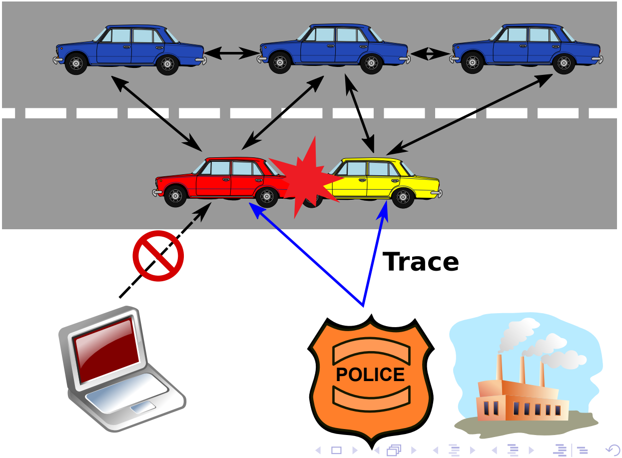


Example

Smart cars

- Authenticity
- Integrity
- Anonymity
- Dynamicity
- Traceability



Motivation

Definition

A *dynamic* group signature allows a member of a group to anonymously sign a message on behalf of the group, and allow new users to join at any time.

Applications: smart cars, control in public transportation, anonymous access control (*e.g.* in public transportation)...

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Main Differences

Static Group	Dynamic Group
GM distributes keys	$\mathcal{U}_i$ makes his secret certified
GM must be trusted	Even colluding **GM**/**OA** cannot sign on behalf of a honest group member
Cannot add new users	

Motivation

Advantages of dynamically growing groups:

- Add users without re-running the **Setup** phase;

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- Add users without re-running the **Setup** phase;
- Even if everyone, including authorities, is dishonest, no one can sign in your name.

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No dynamic group signature scheme based on lattices

Lattice-Based Cryptography

Lattice

A lattice is a discrete subgroup of $\mathbb{R}^n$. Can be seen as integer linear combinations of a finite set of vectors.

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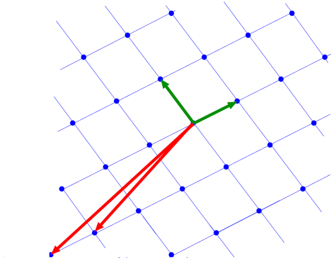

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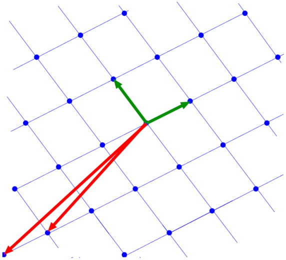

Find a short vector in a lattice is hard.

Lattice-Based Cryptography

Why?

- Simple and efficient;

Lattice-Based Cryptography

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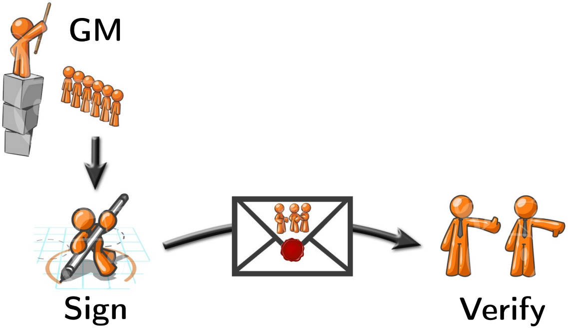
- Secure under worst-case hardness assumptions;
- Powerful functionalities.

Outline

Presentation



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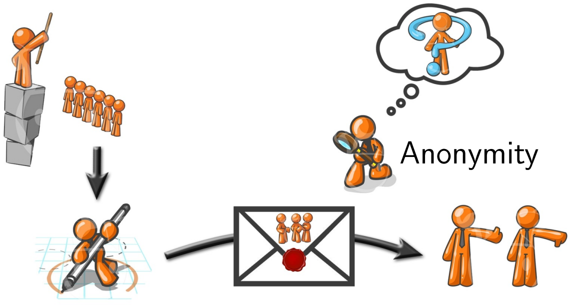

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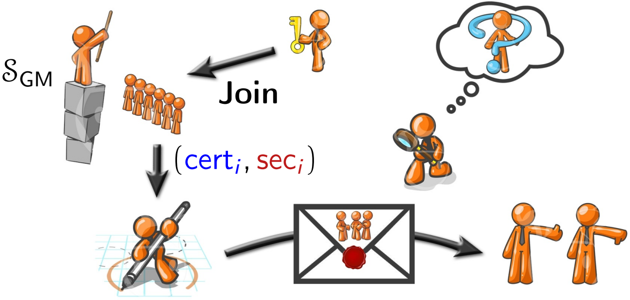

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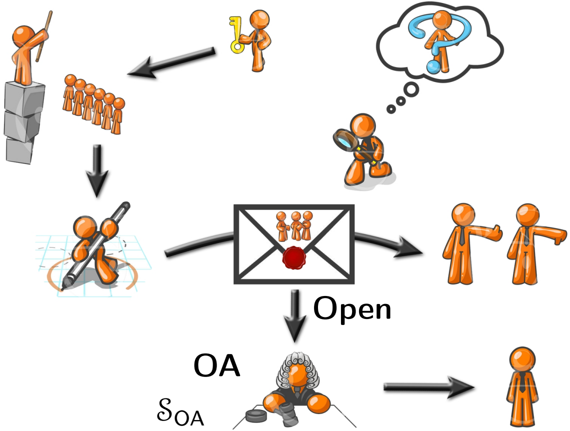

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- **Setup:**
  `Input:` security parameter λ, bound on group size N
  `Output:` public parameters $\mathcal{Y}$, group manager's secret key $\mathcal{S}_{\mathsf{GM}}$, the opening authority's secret key $\mathcal{S}_{\mathsf{OA}}$;

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It is a tuple of algorithms (**Setup**, **Join**, **Sign**, **Verify**, **Open**) acting according to their name.

- **Join:** interactive protocols between $\mathcal{U}_i \rightleftarrows$ **GM**. Provide $(\mathsf{cert}_i, \mathsf{sec}_i)$ to $\mathcal{U}_i$. Where cert_i attests the secret sec_i. Update the user list along with the certificates;

Dynamic Group Signature

Dynamic Group Signature

It is a tuple of algorithms (**Setup**, **Join**, **Sign**, **Verify**, **Open**) acting according to their name.

- **Sign** and **Verify** proceed in the obvious way;
- **Open**:
  `Input`: **OA**'s secret $\mathcal{S}_{\mathsf{OA}}$, M and Σ
  `Output`: i.

Security Notions

Three security notions

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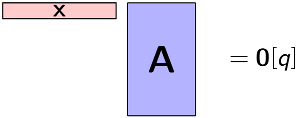
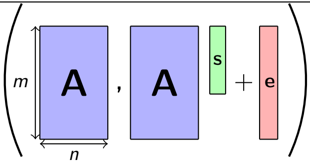
Three security notions

- **Anonymity** Only **OA** can open a signature;
- **Traceability** Security of honest **GM** against malicious users who want to escape from traceability;
- **Non-frameability** Security of honest members against malicious **GM**/**OA** authorities.

Security Assumptions: SIS and LWE

Parameters: n dimension, $m \geqslant n$, q modulus.

For $\mathbf{A} \hookleftarrow \mathbb{Z}_q^{m \times n}$:

Small Integer Solution	**Learning With Errors**
	
	$\mathbf{s} \hookleftarrow \mathbb{Z}_q^n$, $\mathbf{e}$ a small error.
Goal: Given $\mathbf{A} \hookleftarrow \mathbb{Z}_q^{m \times n}$, find $\mathbf{x} \in \mathbb{Z}^m$ small.	Goal: Given $(\mathbf{A}, \mathbf{A}\mathbf{s} + \mathbf{e})$, find $\mathbf{s} \in \mathbb{Z}_q^n$.

Lattice-based cryptography?

Lattice hard problems
find a short vector in a lattice.
Worst-case

↓

Hardness assumptions
LWE, SIS.
Average-case

↓

Security properties
anonymity, traceability, non-frameability.

Outline

From Static to Dynamic

- Designed from a recent static group signature proposed by Ling, Nguyen and Wang [LNW15].

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- Other solutions [GKV10,LLLS13] use membership certificates made of a complete basis...

  ...which is problematic here.

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Difficulties

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- Previous schemes based on [LLLS13] do not interact well with the non-homogeneous terms $\mathbf{v}_i$ needed for *non-frameability* purposes;
- Be secure against *framing attacks* without compromising previous security properties;

From Static to Dynamic Our solution – Ingredients

Boyen's signature (PKC'10)

Given $\mathbf{A} \in \mathbb{Z}_q^{m \times n}$ and $\{\mathbf{A}_i\}_{i=0}^{\ell} \in \mathbb{Z}_q^{m \times n}$, the signature is a **small**

$$\mathbf{d} \in \mathbb{Z}_q^{2m} \;\; \text{s.t.} \;\; \mathbf{d}^T \cdot \left[\frac{\mathbf{A}}{\mathbf{A}_0 + \sum_{i=1}^{\ell} m_i \mathbf{A}_i} \right] = 0[q].$$

The private key is a short $\mathbf{T_A} \in \mathbb{Z}_q^{m \times m}$ s.t. $\mathbf{T_A} \cdot \mathbf{A} = 0[q]$.

In our context: **GM**'s secret is $\mathbf{T_A}$.

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In our context: **GM**'s secret is $\mathbf{T}_{\mathbf{A}}$.

The Böhl *et al.* variant (Eurocrypt'13)

$$\underset{\substack{\uparrow \\ \text{cert}_i}}{\mathbf{d}_i}^T \left[\frac{\mathbf{A}}{\mathbf{A}_0 + \sum_{i=1}^{\ell} m_i \mathbf{A}_i} \right] = \underset{\substack{\uparrow \\ \text{sec}_i}}{\mathbf{z}_i}^T \mathbf{D} + \mathbf{u}^T [q]$$

From Static to Dynamic Our solution

Setup: $\mathcal{Y} = (\mathbf{A}, \{\mathbf{A}_i\}_{i=0}^{\ell}, \mathbf{B}, \mathbf{D}, \mathbf{u})$ $\quad \ell = \log(N)$ ($e.g.$ $\ell = 30$)
Where: $\mathbf{A}, \mathbf{A}_0, \ldots, \mathbf{A}_\ell, \mathbf{B}, \mathbf{D} \in \mathbb{Z}_q^{m \times n}$ and $\mathbf{u} \in \mathbb{Z}_q^n$

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$\mathbf{z}_{i,0} \hookleftarrow$ short vector in $\mathbb{Z}^m$

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	$\xleftarrow{(\mathrm{id}_i, \mathbf{z}_{i,1})}$	$\mathbf{z}_{i,1} \hookleftarrow$ short vector in $\mathbb{Z}^m$
$\mathbf{z}_i = \mathbf{z}_{i,0} + \mathbf{z}_{i,1}$		
$\mathbf{v}_i^T = \mathbf{z}_i^T \mathbf{D}$		
Authenticate $\mathbf{v}_i$, id_i and $\mathbf{z}_i$		

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Authenticate $\mathbf{v}_i$, id_i and $\mathbf{z}_i$ $\xrightarrow{\mathbf{v}_i}$ $\mathbf{d}_i$, s.t.

$$\mathbf{d}_i^T \left[\frac{\mathbf{A}}{\mathbf{A}_0 + \sum_{i=1}^{\ell} \mathrm{id}_i \mathbf{A}_i}\right] = \mathbf{v}_i^T + \mathbf{u}^T [q]$$

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Authenticate $\mathbf{v}_i$, id_i and $\mathbf{z}_i$	$\xrightarrow{\mathbf{v}_i}$	$\mathbf{d}_i$, s.t. $\mathbf{d}_i^T \left[\frac{\mathbf{A}}{\mathbf{A}_0 + \sum_{i=1}^{\ell} \mathrm{id}_i \mathbf{A}_i}\right] = \mathbf{v}_i^T + \mathbf{u}^T \, [q]$
$(\mathrm{cert}_i; \mathrm{sec}_i) = ((\mathrm{id}_i, \mathbf{d}_i); \mathbf{z}_i)$	$\xleftarrow{\mathbf{d}_i}$	

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Sign algorithm:
$\mathbf{c} := \mathbf{Enc}(\mathrm{id}_i, \mathbf{d}_i)$

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$\mathbf{c} := \mathbf{Enc}(\mathrm{id}_i, \mathbf{d}_i)$ $\pi_K :=$ proof that $\mathbf{c}$ is correct and

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Difference with the Ling *et al.* scheme

We encrypt $\mathbf{d}$ and id_i not only id_i to enable signature openings.

From Static to Dynamic Our solution

Open algorithm:

- **OA** decrypts **c** to get $(\mathrm{id}, \mathbf{d})$;
- Using id and **d**, **OA** computes the associated syndrome **v**;
- **OA** checks that $(\mathbf{v}, \mathrm{id}, i, \mathsf{upk}[i], \overbrace{\mathit{sig}}^{=\mathit{Sign}_{\mathbf{usk}[i]}(\mathbf{v}_i, \mathrm{id}_i)})$ is in the records and that *sig* is correct.

  If so then return i; otherwise return $\perp$.

Efficiency

Remark

We use the "smudging" technique: making 2 distributions centered around 0 statistically close using a huge noise.

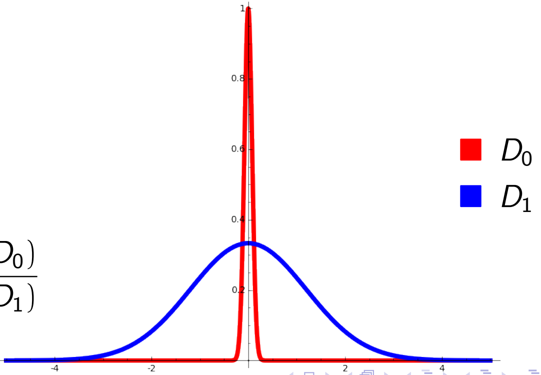
Goal:

$D_0 + D_1 \approx D_1$

in $\mathbb{Z}$ or $\mathbb{R}$

Statistical distance:

$$\Delta(D_0 + D_1, D_1) \approx \frac{\sigma(D_0)}{\sigma(D_1)}$$



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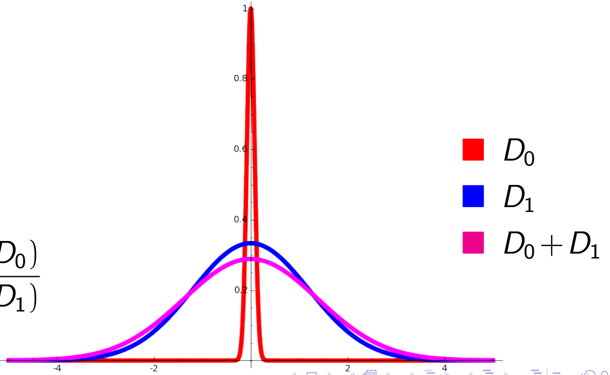

Efficiency

Consequence

We need an exponential-size modulus q in the security parameter λ.

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Problem

Our protocol is somewhat costly.

Outline

Conclusion

Main contribution

First dynamic group signature based on lattice assumptions.

Technical contribution

We combine the Böhl *et al.* variant of Boyen's signature and the Ling *et al.* NIZK proofs.

Extensions

Possible extension supporting proofs of correct opening [BSZ05].
Possible use of the join protocol to certify hidden data.

Open problem

Prove the security without *smudging*: possibly more efficient parameters.

References

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Foundations of group signatures: The case of dynamic groups
(CT-RSA'05)

Aggelos Kiayias and Moti Yung.
Secure scalable group signature with dynamic joins and separable authorities
(International Journal of Security and Networks)

Fabien Laguillaumie, Adeline Langlois, Benoit Libert, Damien Stehlé.
Lattice-based group signature scheme with verifier-local revocation
(Asiacrypt'13)

San Ling, Khoa Nguyen, and Huaxiong Wang.
Group Signatures from Lattices: Simpler, Tighter, Shorter, Ring-Based
(PKC'15)

Question Time

Thank you all for your attention!

One-Time Signature

Definition

A *one-time signature scheme* consists of a triple of algorithms $\Pi^{\mathrm{ots}} = (\mathcal{G}, \mathcal{S}, \mathcal{V})$. *Behaves like a digital signature scheme.*

Strong unforgeability: impossible to forge a valid signature *even for a previously signed message.*

Usage

We use one-time signature to provide CCA anonymity using Canetti-Halevi-Katz methodology.

CCA anonymity

Definition

No PPT adversary $\mathcal{A}$ can win the following game with non negligible probability:

- $\mathcal{A}$ makes open queries.
- $\mathcal{A}$ chooses $M^\star$ and two different $(\mathsf{cert}_i^\star, \mathsf{sec}_i^\star)_{i\in\{0,1\}}$
- $\mathcal{A}$ receives $\sigma^\star = Sign_{\mathsf{cert}_b^\star,\mathsf{sec}_b^\star}(M^\star)$ for some $b \in \{0, 1\}$
- $\mathcal{A}$ makes other open queries
- $\mathcal{A}$ returns b', and wins if $b = b'$

ZK Proofs

Σ-protocol [Dam10]

3-move scheme: (**Commit**, **Challenge**, **Answer**) *between 2 users*.

Fiat-Shamir Heuristic

Make the Σ-protocol non-interactive by setting the challenge to be $H(\mathbf{Commit}, \mathrm{Public})$

Smudging